UITWERKING Toets QF II, 18 mei 1999

1 a) $\psi(1,2) = \psi_1 \psi_2 = \sqrt{\frac{1}{4\pi^2}}\, e^{in_1\phi_1} e^{in_2\phi_2}$

(hiermee kunnen alle eigenfuncties geconstrueerd worden:
bv. $\psi_1+\psi_2$: algemene opl. $\sum_{nm} c_{nm}\psi_n\psi_m$, neem
combinatie $n=0, m=m$ en $n=n, m=0 \Rightarrow \psi_m+\psi_n$)

$E = E_1 + E_2 = (n_1^2+n_2^2)\frac{\hbar^2}{2ma^2}$

b) $\sqrt{\frac{2}{3\pi^2}}\cos^2\phi_1\, e^{i\phi_2} = \sqrt{\frac{2}{3\pi^2}}\left(\tfrac{1}{2}\right)^2\left(e^{i\phi_1}+e^{-i\phi_1}\right)^2 e^{i\phi_2} =$

$\sqrt{\frac{2}{3\pi^2}}\,\frac{1}{4}\left(e^{2i\phi_1}+2+e^{-2i\phi_1}\right)e^{i\phi_2}$

$\downarrow$ $E=\frac{4\hbar^2}{2ma^2}$ $\quad$ $\downarrow$ $E=0$ $\quad$ $\downarrow$ $E=\frac{4\hbar^2}{2ma^2}$ $\quad$ $\searrow$ $E=\frac{\hbar^2}{2ma^2}$

$\left[\begin{array}{l}\psi(t)=e^{-iHt/\hbar}\psi(0)\\ \overset{*}{=} e^{-iEt/\hbar}\psi(0)\\ *\text{ als } \psi \text{ is eigenfunctie van } H\end{array}\right]$

$t=T$

$\psi(T) = \sqrt{\frac{2}{3\pi^2}}\,\frac{1}{4}\left(e^{2i\phi_1}e^{-i\frac{4\hbar^2}{2ma^2}\frac{T}{\hbar}}+2+e^{-2i\phi_1}e^{-i\frac{4\hbar^2}{2ma^2}\frac{T}{\hbar}}\right)e^{i\phi_2}e^{-i\frac{\hbar^2}{2ma^2}\frac{T}{\hbar}}$

normering: $\int_0^{2\pi}|\psi(T)|^2 dr = \frac{2}{3\pi^2}\frac{1}{4^2}\left((2\pi)^2+4(2\pi)^2+(2\pi)^2\right)=1$

c) $\psi(T) = \sqrt{\frac{2}{3\pi^2}}\,\frac{1}{4}\left(e^{2i\phi_1}e^{-i\frac{5\hbar^2}{2ma^2}\frac{T}{\hbar}}+2e^{-i\frac{\hbar^2}{2ma^2}\frac{T}{\hbar}}+e^{-2i\phi_1}e^{-i\frac{5\hbar^2}{2ma^2}\frac{T}{\hbar}}\right)e^{i\phi_2}$

$\downarrow$ $E=5$ $\quad$ $\downarrow$ $E=1$ $\quad$ $\downarrow$ $E=5$

kans op $E=5$: $\frac{2}{3\pi^2}\frac{1}{4^2}\left((2\pi)^2+(2\pi)^2\right)=\frac{1}{3}$

kans op $E=1$: $\frac{2}{3\pi^2}\frac{1}{4^2}\left(4(2\pi)^2\right)=\frac{2}{3}$

2 a) $L^2 Y_{\ell,m} = \ell(\ell+1)\hbar^2 Y_{\ell,m}$ $\qquad$ Let op: $\frac{1}{\sqrt{8\pi}} \sim Y_{0,0}$

$\Rightarrow L^2 Y_{0,0}=0$, $L^2 Y_{1,m} = 1(1+1)\hbar^2 Y_{1,m} = 2\hbar^2 Y_{1,m}$

$Y_{\ell m}$ zijn orthonormaal

b) $\ell=0$ deel $\int\left(\frac{1}{\sqrt{8\pi}}\right)^2 d\Omega = \frac{1}{8\pi}\cdot 4\pi = \frac{1}{2}$ $\qquad$ $\langle L^2\rangle = \frac{1}{2}\times 0 = 0$

$\ell=1$ deel $\int\left(\frac{1}{\sqrt{10}}\left[-iY_{1,1}+\sqrt{7}Y_{1,0}+Y_{1,-1}\right]\right)^2 d\Omega$

$= \frac{1}{10}(1+7+1)=\frac{1}{2}$ $\qquad$ $\langle L^2\rangle = \frac{1}{2}\cdot\ell(\ell+1)=\frac{1}{2}\cdot 2=1$

$\Rightarrow$ TOTAAL $\langle L^2\rangle = 1$

c) $L_z Y_{\ell,m} = m\hbar Y_{\ell,m} \Rightarrow L_z = -1, 0, 1$

d) $\langle L_z\rangle = \frac{1}{2}\times 0 + \frac{1}{10}\times 1 + \frac{7}{10}\times 0 + \frac{1}{10}(-1) = 0$

e) $L_z = 0$: $\psi = N\left(\frac{1}{\sqrt{8\pi}}+\sqrt{\frac{7}{10}}Y_{1,0}\right)$

integreren over de ruimte $\int|\psi|^2 d\Omega = N^2\left(\frac{1}{2}+\frac{7}{10}\right) = N^2\frac{16}{10} \equiv 1$

$N=\sqrt{\frac{10}{16}}$